APPLICATION NO. 09/826,118

TITLE OF INVENTION: Wavelet Multi-Resolution Waveforms

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Clean version of how the CLAIMS will read.

APPLICATION NO. 09/826,118
INVENTION: Multi-Resolution Waveforms
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CLAIMS

WHAT IS CLAIMED IS:

Claim **1.** (currently amended) An iterative eigenvalue least-squares LS method for designing digital mother Wavelets at baseband for multi-resolution waveforms and filters, said method comprising steps:

power spectral density PSD representative requirements for said mother Wavelet ψ frequency ω response $\psi_\omega(\omega)$ in a multi-channel filter bank, specify

a) passband frequency range for waveform transmission,
b) stopband spacing between adjacent filters,
c) bounds on ripple over said passband,
d) stopband filter attenuation,
e) rolloff with frequency outside stopband,
f) quadrature mirror filters QMF require the sum of said PSD's for contiguous filter responses to be flat over deadband which is said stopband,
g) symbol-to-symbol interference ISI,
h) adjacent channel interference ACI,

said LS error metrics to measure said requirements (a)-(h) are derived as functions of said Wavelet $\psi(n)$ assuming

i) T and 1/T are sample interval and sample rate equal to Nyquist sample rate,
j) ψ is real and symmetric about n=0,
k) n= 0, +/-1, . . ., +/-ML/2 digital index over said ψ,
l) M is interval between contiguous said ψ,

m) $1/MT$ is said ψ symbol rate and channel-to-channel separation,

n) L is length of said ψ in units of said M,

said multiple-resolution properties require said LS metrics to be constructed as functions of said Wavelet Fourier harmonics $\psi_k(k)$ with $k=0, +/-1, \ldots, +/-(N_k-1)$ and $N_k \geq L$ is a design parameter,

it is sufficient to use positive $n=0,1,\ldots, ML/2$ and $k=0,1,\ldots, N_k-1$ since said $\psi(n)$ and $\psi_k(k)$ are real and symmetric,

$ML/2+1 \times N_k$ matrix bw wherein "x" reads "by" maps $\psi_k(k)$ into $\psi(n)$ to within a scale factor by equations

$$\psi(n) = \sum_k bw(n+1,k+1)\psi_k(k) \quad \text{for } n\geq 0,\ k\geq 0,$$

$$\begin{aligned} bw(n+1,k+1) &= 1 && \text{for } n=0, \\ &= 2\cos(2\pi\, nk/ML) && \text{otherwise}, \\ &= \text{row } n+1, \text{ column } k+1 \text{ element of bw}, \end{aligned}$$

said $\psi(n)$ and $\psi_k(k)$ are converted to column vectors by equations

$h = ML/2+1 \times 1$ column vector of $\psi(n)$ with elements

$$h(1) = \psi(n=0),$$

$$h(n+1) = 2\psi(n) \quad \text{for } n=1,2,\ldots ML/2,$$

$h_k = N_k \times 1$ column vector of $\psi_k(k)$ with elements

$$h_k(1) = \psi_k(k=0),$$

$$h_k(k+1) = 2\psi_k(k) \quad \text{for } k=1,2,\ldots, N_k-1,$$

said bw maps h_k into h to within a scale factor by matrix equation

$$h = bw\ h_k,$$

LS error metrics for band = passband, stopband, and QMF deadband requirements are derived as quadratic forms in said h,

said LS error metrics are converted by said bw mapping into quadratic forms in said h_k equal to $J(\text{band})=h_k{}' R\ h_k$ wherein said $h_k{}'$ is the transpose of h_k and said R is a real square symmetric matrix of LS errors in meeting said requirements.

LS ISI,ACI error metrics J(ISI),J(ACI) are derived as non-linear

quadratic forms in h and converted by said bw matrix to the non-linear quadratic form in h_k equal to $J(ISI)=\delta E'\delta E$, $J(ACI)=2\delta E'\delta E$ wherein $\delta E = AHh_k$ is a column vector and matrix "A" in the matrix product AH is a function of said h hereby introducing said non-linearity, and said AH differ for ISI and ACI error metrics,

LS cost function J is the weighted sum of said LS error metrics

$$J = \sum w(\text{LS metric})\ J(\text{LS metric})$$

with summation over said LS metrics= passband, stopband, QMF deadband, ISI, ACI with normalized weights

$$\sum w(\text{LS metric})=1,$$

said weights are free design parameters,

said iterative eigenvalue LS algorithm at each step finds the optimum eigenvalue and eigenvector which minimize said quadratic form J in h_k for a constant said "A",

said eigenvector is the optimum h_k which minimizes said J and said bw equation derives the corresponding optimum h which minimizes said J,

step 1 in said iterative algorithm finds said optimum eigenvalue, eigenvector, h_k, h of J reduced by deleting said non-linear ISI and ACI LS quadratic error metrics,

said h is used to evaluate said "A" matrices for step 2,

step 2 finds said optimum eigenvalue, eigenvector, h_k, h for minimum J using said "A" from step 1,

said h is used to evaluate said "A" for step 3,

steps 3,4, etc. continue until said minimum J converges to a steady value and,

said optimum $\psi_k(k)$ uses said bw to calculate optimum $\psi(n)$ for implementation as said Wavelet FIR digital waveform and filter time response.

Claim **2.** (currently amended) An LS method for designing digital mother Wavelets at baseband for multi-resolution waveforms and filters, said method comprising steps:

said PSD waveform representative requirements and assumptions are recited in (a)-(n) in Claim **1**,

said multiple-resolution properties require said LS metrics to be constructed as functions of said $\psi_k(k)$,

said LS error metrics for said passband, stopband, and QMF deadband requirements are derived as squared vector norm functions of said h and converted by said bw matrix into $J(\text{band}) = \|Bh_k\|^2$ wherein $\|Bh_k\|$ is the vector norm of the column vector Bh_k and said B is the matrix of LS errors in meeting said requirements and wherein said squared vector norm is suitable for LS optimization,

LS ISI,ACI error metrics J(ISI),J(ACI) are derived as squared vector norm functions equal to $J(ISI=\|\delta E\|^2$, $J(ACI)=2\|\delta E\|^2$ using said column vectors $\delta E = AHh_k$ in claim **1**,

LS cost function J is said weighted sum of said LS error metrics equal to $J=\sum w(\text{LS metric})J(\text{LS metric})$ defined in claim **1**,

an LS gradient search algorithm finds optimum $h_k(k)$ to minimize J,

step 1 of said LS gradient search algorithm uses a Remez-exchange algorithm to find said optimum $h_k(k)$ for said J reduced to said passband and stopband LS metrics,

step 2 uses the estimated $h_k(k)$ from step 1 to initialize said gradient search,

step 3 selects one of several available gradient search algorithms, gradient search parameters, and stopping rules,

step 4 implements said algorithm, parameters, and stopping rule selected in step 3 to derive said optimum $h_k(k)$ to minimize J and,

said optimum h_k uses said bw to calculate optimum $\psi(n)$ for

implementation as said Wavelet FIR digital waveform and filter time response.

Claim **3**. (currently amended) Wherein said mother Wavelet in claims **1,2** generates multi-resolution dilated Wavelets, comprising steps and design:

said Wavelet parameters are

a) scaling parameter p dilates sampling by factor 2^p equivalent to sub-sampling by factor 2^p,

b) translation parameter q translates said ψ by qM digital samples,

c) frequency offset k is set by design,

d) symbol repetition interval said M remains constant,

e) Wavelet length said L in units of said M remains constant,

step 1 uses said design harmonics ψ_k to generate said FIR time response $\psi(n_p)$ at baseband with said bw equation

$$\psi(n_p) = \sum_k bw(n_p+1, k+1)\ \psi_k(k)$$

recited in claim **1** with

$$bw = 2\cos(2\pi\ n_p k/ML) \text{ for } n_p>0$$

wherein $n_p=n/2$^p is n sub-sampled or equivalently dilated by the factor 2^p,

step 2 uses said $\psi(n_p)$ to construct said multi-resolution Wavelet $\psi_{p,q,M,L,k}$ with equation

$$\psi_{p,q,M,L,k} = 2\text{^}(-p/2)\ \psi(n_p-qM)\ \exp(j2\pi\ kn_p/ML)$$

which is said FIR time response for parameters p,q,M,L,k wherein the subset p,M,L are the scale parameters,

design of said multi-resolution Wavelet includes

f) said T for n is increased to T2^p for n_p,

g) said 1/T is reduced to 1/T2^p,

h) said ψ symbol rate 1/MT equal to said channel-to-channel separation is reduced to 1/MT2^p in Hz and,

i) said ψ length (ML+1)T in seconds is stretched to (ML+1)T2^p in seconds.

Claim **4**. (currently amended) Wherein said mother Wavelet in claims **1,2** generates multi-resolution constant sample rate dilated Wavelets, comprising steps and design:

said Wavelet parameters are

a) said p dilates said ψ to increase said length from ML+1 to M_pL+1 where M_p=M2^p is the dilated interval between contiguous ψ's,

b) said q translates said ψ by qM_p digital samples,

c) said k is set by design,

d) said M_p=M2^p is dilated M,

e) said L remains constant,

step 1 uses said design harmonics ψ_k to generate said FIR time response $\psi(n_p)$ at baseband with said bw equation

$$\psi(n) = \sum_k bw(n+1,k+1)\ \psi_k(k)$$

recited in claim **1** with

$$bw = 2\cos(2\pi\ nk/M_pL) \text{ for } n>0,$$

step 2 uses said $\psi(n)$ to construct said multi-resolution Wavelet $\psi_{p,q,M,L,k}$ with equation

$$\psi_{p,q,M,L,k} = 2^{\wedge}(-p/2)\ \psi(n-qM_p)\ \exp(j2\pi\ kn/M_pL)$$

which is said FIR time response for parameters p,q,M,L,k,

design of said multi-resolution Wavelet includes

f) said T remains constant,

g) said 1/T remains constant,

h) said ψ symbol rate 1/MT equal to said channel-to-channel separation is reduced to $1/M_pT$=1/MT2^p in Hz and,

i) said ψ length (ML+1)T in seconds is stretched to (ML2^p+1)T in seconds.

Claim **5**. (currently amended) Wherein said mother Wavelet in claims **1,2** generates multi-resolution up-sampled Wavelets, comprising steps and design:

said Wavelet parameters are

a) said p up-samples said digital sampling rate 1/T to 2^p/T,

b) said q translates said ψ by qM digital samples,

c) said k is a design parameter,

d) said M is constant

e) said L is constant

step 1 uses said design harmonics h_f, ψ_k to generate said FIR time response $\psi(n_p)$ at baseband with said equation

$$\psi(n_p) = \sum_k bw(n_p+1,k+1)\ \psi_k(k)$$

recited in claim 1 with

$$bw = 2\cos(2\pi\ n_p k/ML) \text{ for } n_p>0$$

wherein n_p is n up-sampled by the factor 2^p and defined by equations

$$n_p = n\_p + n\ 2\text{^}p$$

$$n\_p = 0,1,2,\ldots,2\text{^}p-1$$

wherein n_p is the index over the additional samples added to each sample n by said up-sampling,

step 2 uses said $\psi(n_p)$ to construct said multi-resolution Wavelet $\psi_{p,q,M,L,k}$ with equation

$$\psi_{p,q,M,L,k} = 2\text{^}(-p/2)\ \psi(n_p-qM)\ \exp(j2\pi\ kn_p/ML)$$

which is said FIR time response for parameters p,q,M,L,k,

design of said multi-resolution Wavelet includes

f) said T is decreased to T/2^p,

g) said 1/T is increased to 2^p/T,

h) said ψ symbol rate 1/MT equal to said channel-to-channel separation is increased to 2^p/MT in Hz and,

i) said ψ length (ML+1)T in seconds is reduced to (ML +1)T/2^p in seconds.

Claim **6** (currently amended) Wherein said multi-resolution Wavelets in Claims **1-5** have properties comprising:

said scale parameters p,M,L and said design parameter 1/T specify said multi- resolution Wavelets at baseband and said q,k specifify time,frequency translations from baseband,

said design harmonics $\psi_k(k)$ of mother Wavelet are said design coordinates for multi-resolution Wavelets,

said design harmonics $\psi_k(k)$ use said bw matrix to generate said multi-resolution Wavelet baseband time response $\psi(n)$ for dilation, dilation of Wavelet length, and up-sampling as recited in Claims **3-5** and which is translated in time and frequency to said multi-resolution Wavelet $\psi_{p,q,M,L,k}$,

said design harmonics $\psi_k(k)$ are few in number compared to said $\psi(n)$,

said ψ is designed to support a bandwidth-time product $B_fT=1+\alpha$ with no zero excess bandwidth $\alpha=0$,

said multi-resolution Wavelets are designed to behave like an accordion in that at different scales said Wavelets are stretched and compressed versions of the mother Wavelet with appropriate time and frequency translation,

said optimization techniques in claims **1,2** assume said $\psi(n)$ symmetric about n=0 and are applicable to other arrangements of $\psi(n)$ with self-evident modifications,

optimization algorithms for finding said optimum set of $\psi_k(k)$ use linear LS waveform and filter design methods recited in claims **1,2** and also use other methods and,

said linear waveform and filter LS design methods can be modified to design waveforms for applications including bandwidth efficient modulation BEM and synthetic aperture radar SAR.